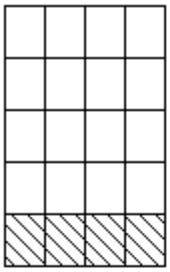
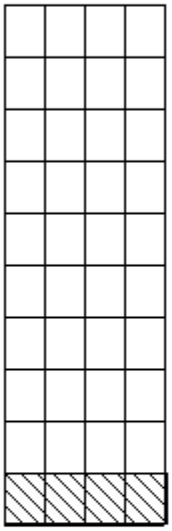
A 20oz container is $\frac{1}{5}$ OJ. Another container is 40oz and is $\frac{1}{10}$ OJ. When the two are mixed, what's the fraction of OJ in the mixture?



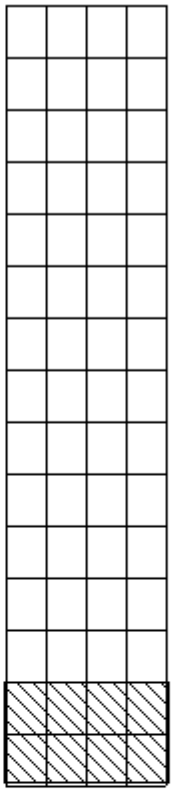
$\frac{1}{5}(20\text{oz}) = \frac{1}{5}\cdot 5\cdot 4\text{oz} = 4\text{oz}\cdot\text{OJ}$

Cross off the 5's. So this container has 4 oz of OJ in it.



$\frac{1}{10}(40\text{oz}) = \frac{1}{10}\cdot 10\cdot 4\text{oz} = 4\text{oz}\cdot\text{OJ}$

Cross of the 10's. So this container also has 4 oz OJ in it.



When we combine the two containers, we see that the concentration of OJ is $\frac{8}{60}$

Equation Approach
Define x to be the concentration of OJ in the final mixture.

$$\frac{1}{5}\cdot 20 + \frac{1}{10}\cdot 40 = x\cdot 60$$

$$4 + 4 = x\cdot 60$$

$$8 = x\cdot 60$$

$$x = \frac{8}{60}$$

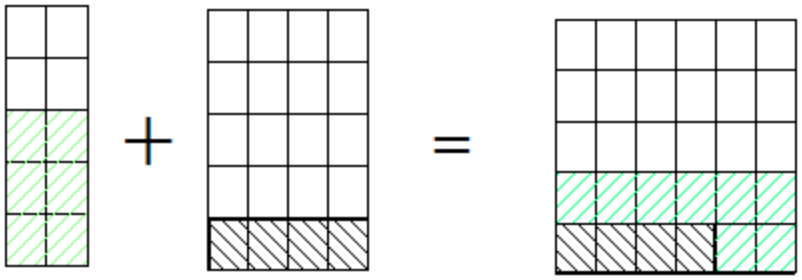
You Try:

A 20oz container is $\frac{1}{10}$ OJ. Another container is 50oz and contains $\frac{1}{5}$ OJ. Find the concentration of OJ in the final mixture.

1) Draw a picture to represent the first container.
2) Mark the concentration of OJ in the first container.
3) Draw a picture to represent the second container.
4) Mark the concentration of OJ in the second container.
5) Draw the two containers put together.
6) Mark the concentration of OJ in the combined container.
7) Confirm the steps outlines above by setting up and solving an equation.

+

A 10oz container has an unknown concentration of OJ. Another container is 20oz and is $\frac{1}{5}$ OJ. When the two are mixed, the fraction of OJ is $\frac{1}{3}$. Find the concentration of OJ in the 10oz container.



10 oz

20 oz total

$\frac{1}{5}$(20oz)=4oz OJ

30oz total

$\frac{1}{3}$(30oz)=10 oz OJ

When we combine the two containers, we see that the concentration of OJ is $\frac{8}{60}$

If the final amount of OJ is 10oz, and there are 4 oz in the second container, then there must be 6 oz in the first container. So that concentration is $\frac{6}{10}$

Equation Approach
Define x to be the concentration of OJ in the first container.

$$x\cdot(10) + \frac{1}{5}\cdot(20) = \frac{1}{3}(30)$$

$$10x + 4 = 10$$

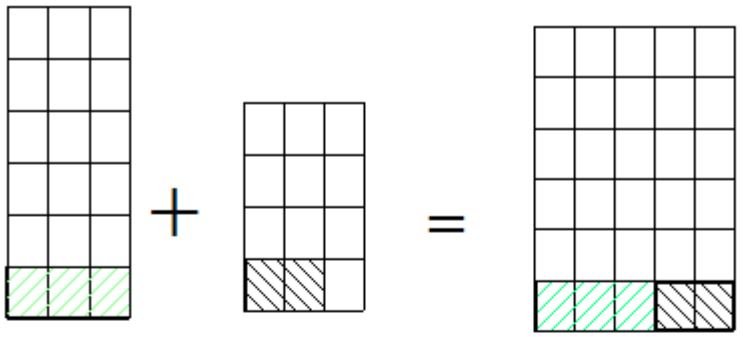
$$10x = 6$$

$$x = \frac{6}{10}$$

A 20oz container has an unknown concentration of OJ. Another container is 40oz and is $\frac{1}{5}$ OJ. When the two are mixed, the fraction of OJ is $\frac{1}{4}$. Find the concentration of OJ in the 20oz container.

1) Draw a picture to represent the first container.
2) Draw a picture to represent the second container.
3) Mark the concentration of OJ in the second container.
4) Draw the two containers put together.
5) Mark the concentration of OJ in the combined container.
6) Using the picture that represents the the concentration in the combined container, mark the appropriate concentration of OJ in the first container.
7) Setup and solve an equation to confirm the concention you have found using pictures.
8) Do you truly understand what you have just done?

A 18oz container is $\frac{1}{6}$ OJ. Another container is 12oz and the concentration of OJ is not known. two are mixed, the fraction of OJ is $\frac{1}{6}$. Find the concentration of OJ in the 12oz container.



18 oz

$\frac{1}{6}$(18oz)=3oz OJ

12 oz total

30oz total

$\frac{1}{6}$(30oz)=5 oz OJ

If the final amount of OJ is 30oz, and there are 3oz OJ oz in the first container, then there must be 2 oz OJ in the first container. So that concentration is $\frac{2}{12}$

Equation Approach
Define x to be the concentration of OJ in the second container.

$$\frac{1}{6}\cdot(18) + x\cdot(12) = \frac{1}{6}(30)$$

$$3 + 12x = 5$$

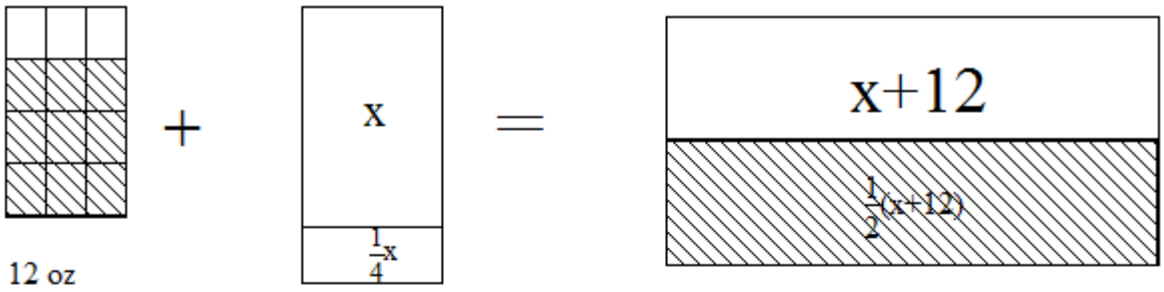
$$12x = 2$$

$$x = \frac{6}{12}$$

A 20oz container is $\frac{1}{10}$ OJ. Another container is 40oz and the concentration of OJ is not known. When the two are mixed, the fraction of OJ is $\frac{1}{4}$. Find the concentration of OJ in the 40oz container.

1) Draw a picture to represent the first container.
2) Mark the concentration of OJ in the first container.
3) Draw the second container.
4) Draw the two containers put together.
5) Mark the concentration of OJ in the combined container.
6) Using the picture that represents the the concentration in the combined container, mark the appropriate concentration of OJ in the second container.
7) Setup and solve an equation to confirm the concention you have found using pictures.
8) Do you truly understand what you have just done?

There are two containers. The first is 12oz and is $\frac{3}{4}$ OJ. The second is $\frac{1}{4}$ OJ. When the two are mixed, the result is $\frac{1}{2}$ OJ. Find the size of the second container.



12 oz

$\frac{3}{4}(12oz)=\frac{3}{4}(4\times3oz)$

$=3\times3oz=9oz$ OJ

Equation Approach
Define x as the size of container two.

$$\frac{3}{4}(12) + \frac{1}{4}x = \frac{1}{2}(x + 12)$$

In words, this equation says that the OJ in the two containers on the left must be equal to the OJ in the containers once they're added and mixed.

$$\frac{3}{4}\cdot4\cdot3 + \frac{1}{4}x = \frac{1}{2}x + \frac{1}{2}\cdot12$$

$$9 + \frac{1}{4}x = \frac{1}{2}x + 6$$

$$9 - 6 = \frac{1}{2}x - \frac{1}{4}x$$

$$3 = \frac{2}{4}x - \frac{1}{4}x$$

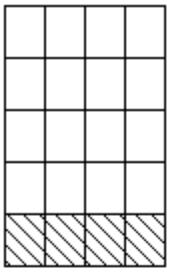
$$3 = \frac{1}{4}x$$

$$4\cdot3 = \frac{1}{4}x\cdot4$$

There are two containers. The first is 15oz and is $\frac{3}{5}$ OJ. The second is $\frac{1}{6}$ OJ. When the two are mixed, the result is $\frac{1}{4}$ OJ. Find the size of the second container.

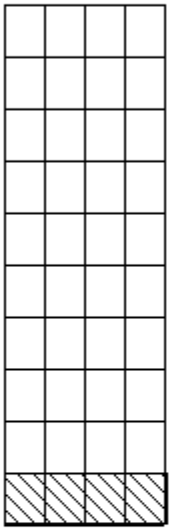
1) Draw a picture of the first container
2) Mark the concentratio of OJ in the first container.
3) Draw a picture of the second container, but do now draw grids because you don't know the number of oz that this container is. Mark a reasonable portion to represent $\frac{1}{6}$ of the container.
4) Draw the combined container, and mark a reasonable portion to represent the combined concentration of $\frac{1}{4}$.
5) Setup and solve an equation to find the size of the second container. Be sure to check by drawing a careful picture, as in the problems above.

Is the concentration of OJ in the final mixture a simple average of the two individual concentrations?



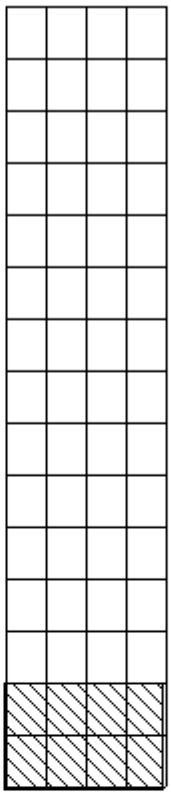
$$\frac{1}{5}(20\text{oz}) = \frac{1}{5}\cdot 5\cdot 4\text{oz} = 4\text{oz}\cdot\text{OJ}$$

Cross off the 5's. So this container has 4 oz of OJ in it.



$$\frac{1}{10}(40\text{oz}) = \frac{1}{10}\cdot 10\cdot 4\text{oz} = 4\text{oz}\cdot\text{OJ}$$

Cross of the 10's. So this container also has 4 oz OJ in it.



When we combine the two containers, we see that the concentration of OJ is $\frac{8}{60}$

Equation Approach
Define x to be the concentration of OJ in the final mixture.

$$\frac{1}{5}\cdot 20 + \frac{1}{10}\cdot 40 = x\cdot 60$$

$$4 + 4 = x\cdot 60$$

$$8 = x\cdot 60$$

$$x = \frac{8}{60}$$

Arithmetic average of the two concentrations.

$$\frac{\frac{1}{5} + \frac{1}{10}}{2} = \frac{\frac{2}{10} + \frac{1}{10}}{2} = \frac{\frac{3}{10}}{2} = \frac{3}{10\cdot 2} = \frac{3}{20}$$

Convert this to 60ths.

$$\frac{3}{20}\cdot\frac{3}{3} = \frac{9}{60}$$

This result is clearly different, so the concentation is not the ARITHMETIC mean of the two individual concentrations.

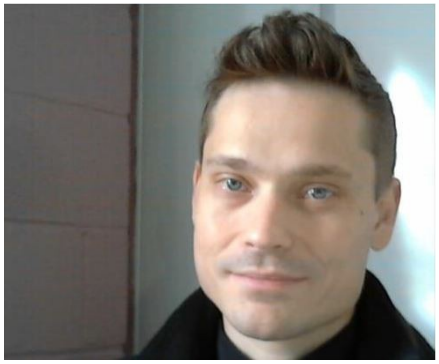
Given a container of size x, with a concentration of OJ c_1, and a container of size d, with a concentration of OJ c_2, find an expression for the concentration of OJ in the final mixture.

$x \cdot c_1 + d \cdot c_2 = z(x + d)$

This equation states that the juice in the individual containers is the same as the juice in the combined containers.

Now just divide both sides by x+d.

$$\frac{x \cdot c_1 + d \cdot c_2}{x + d} = z$$



I hope this picture approach has been truly insightful and helpful. If you spot an error, please email me at towsiak@yahoo.com
Tom
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