

1) $r = -3\cos(\theta)$

Setup the problem

2) $r \cdot r = -3r \cdot \cos(\theta)$

Multiply both sides by r

$$r^2 = -3r\cos(\theta)$$

3) $x^2 + y^2 = -3x$

Replace r^2 with $x^2 + y^2$ and $r\cos(\theta)$ with x

4) $x^2 + y^2 + 3x = 0$

Move $-3x$ to the left by adding it

5) $x^2 + 3x + y^2 = 0$

Rearrange so the x terms are together

6) $\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + y^2 = 0$

Complete the square on the terms with x

7) $\left(x + \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$

Move the $-\frac{9}{4}$ from step 6) to the right side

8) $\left[x - \left(-\frac{3}{2}\right)\right]^2 + y^2 = \left(\frac{3}{2}\right)^2$

Rewrite as shown so you can easily identify the horizontal shift, and the radius for the circle. Remember that something like $x+3$ is the same as $(x-(-3))$.

Now we can conclude we have a circle with center $\left(-\frac{3}{2}, 0\right)$ and radius $r = \frac{3}{2}$

