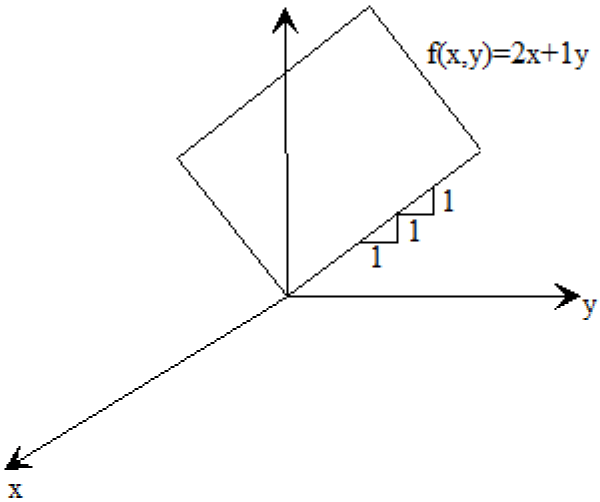
1)

$f(x,y)=2x+1y$

$f_y(x,y)=1=\frac{1}{1}$

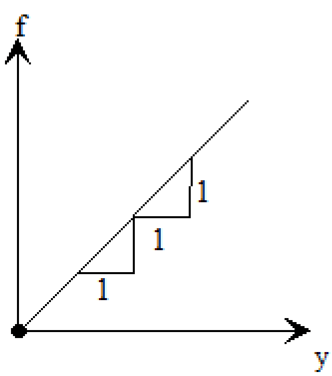
Below is a plot of the plane this function represents.

To see the meaning of $\frac{1}{1}$ graphically, look at the staircase along the y-axis. You see that every time y increases by 1, f also increases by 1. This is the simple meaning of this number.



Below is a graph of $f(0,y)=2(0)+1y=y$

This tells us that along x=0, the function is just y.



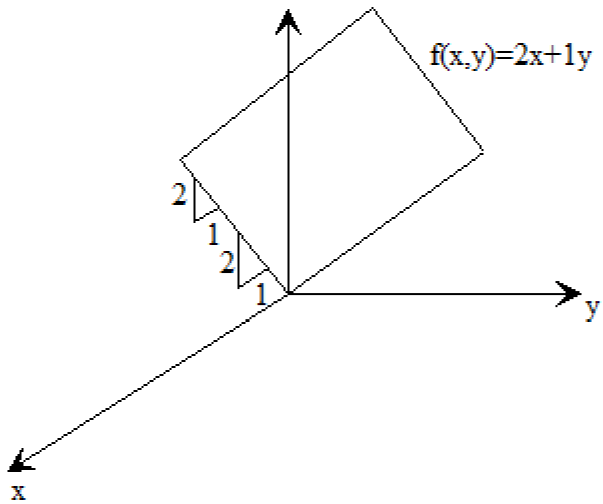
2)

$f(x,y)=2x+1y$

$f_x(x,y)=2=\frac{2}{1}$

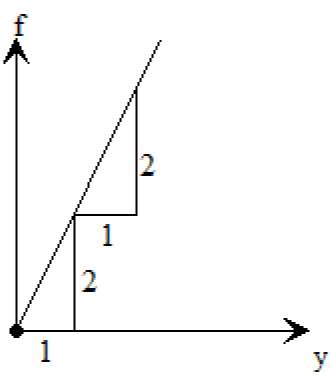
Below is a plot of the plane this function represents.

To see the meaning of $\frac{2}{1}$ graphically, look at the staircase along the x-axis. You see that every time x increases by 1, f increases by 2. This is the simple meaning of this number.



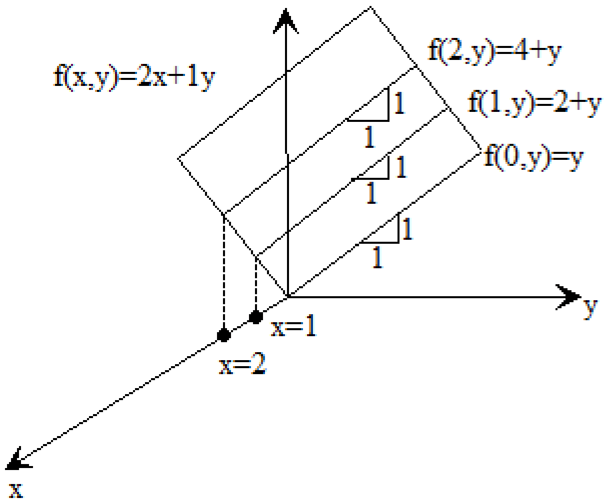
Below is a graph of $f(x,0)=2x+1(0)=2x$

This tells us that along y=0, the function is just 2x.



3)

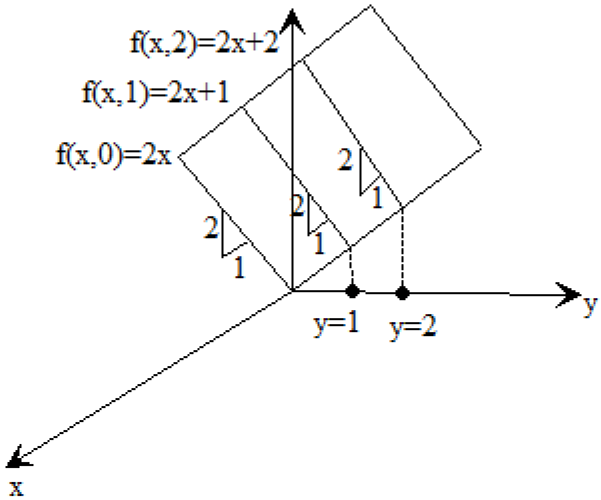
Now we ask whether the slope changes as we change x. The answer to this question is no. Look at the little slope triangles for each choice of x. Clearly, they are the same.



This allows us to conclude that the slope in the y direction does not depend on the value of x.

4)

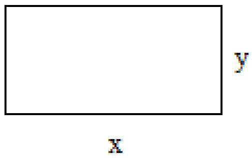
Now we ask whether the slope changes as we change x. The answer to this question is no. Look at the little slope triangles for each choice of x. Clearly, they are the same.



This allows us to conclude that the slope in the x direction does not depend on the value of y.

5)

Now we're going to give these a physical interpretation. Below is a rectangle. We write a function for the perimeter. This amounts to summing the x's and the y's.

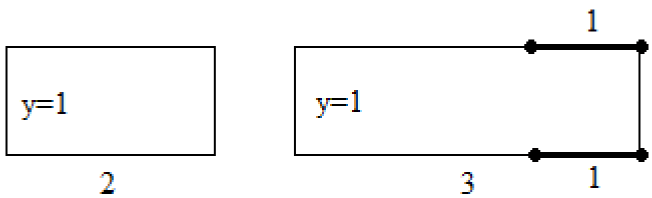


So the function is $P(x,y)=x+y+x+y=2x+2y$

The rate of change of the perimeter with respect to x is

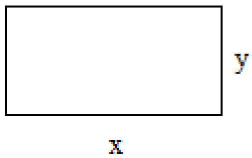
$f_x(x,y)=2=\frac{2}{1}$

This means that every time x increases by 1, the length of the perimeter increases by 2. But this 2 is really 1+1, and to be more exact, it's $\frac{1}{1}+\frac{1}{1}$. What is this? This is a sum of two simple rates. It's illustrated below. We fix y at 1, and let x change.



So remember that $\frac{2}{1}=\frac{1}{1}+\frac{1}{1}$

6) Below is a rectangle. We write a function for the perimeter. This amounts to summing the x's and the y's.

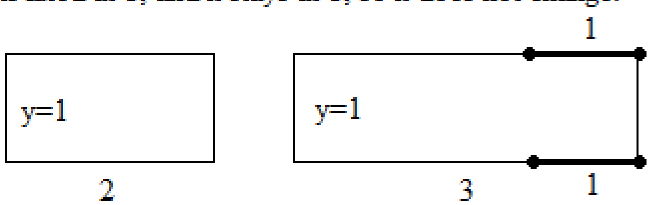


So the function is $P(x,y)=x+y+x+y=2x+2y$

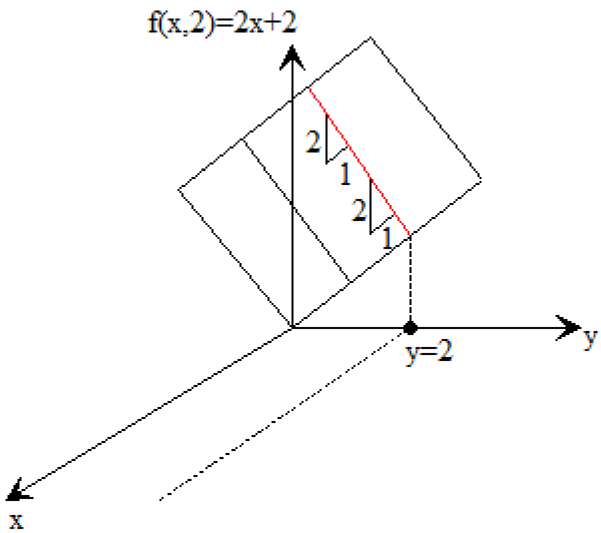
The rate of change of the perimeter with respect to x is

$f_x(x,y)=2=\frac{2}{1}$

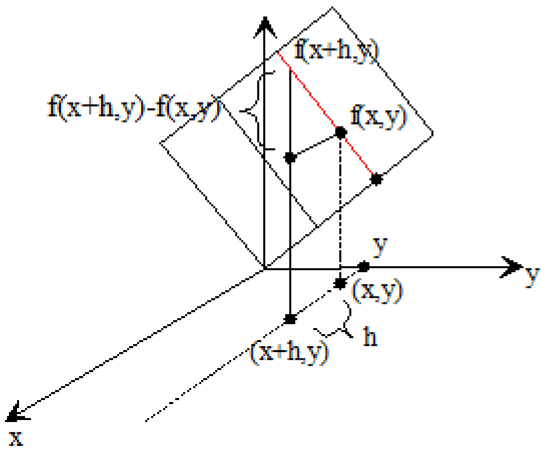
Where does the 2y go exactly? Well, as you can see in the picture below, when we change the x, the y is treated as a constant. This means it does not change. If something doesn't change, it's rate of change is 0. In this case, the y is fixed at 1, and it stays at 1, so it does not change.

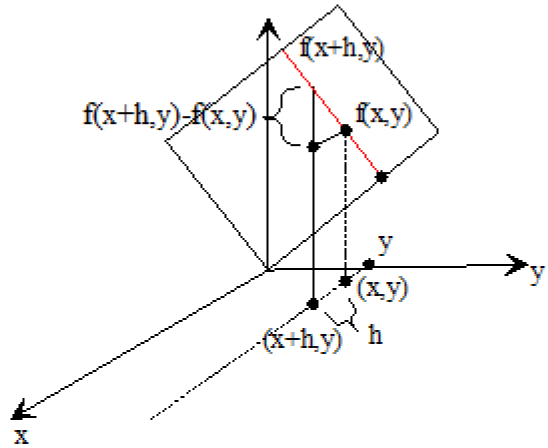


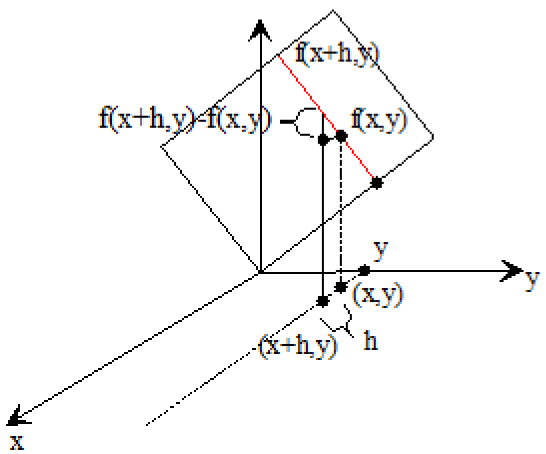
You can see a graph of the function $f(x,y)=2x+y$ below. Do you see how moving along the red line keeps y fixed at 2? This means it does not change. So the rate of change of f with respect to x is 2, and the term "y" doesn't change, so it's 0.



7) Now let's take a look at a more mathematical version. Here, remember, y is fixed. We write y, but we mean this letter to represent a constant with respect to x. So the x changes by the amount h, but the y does not change.







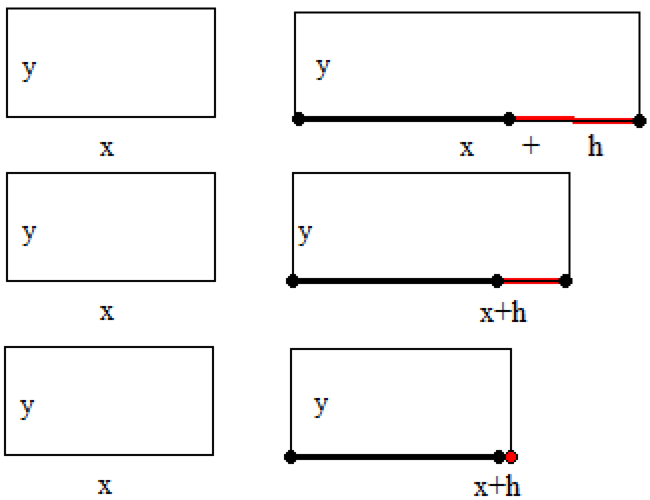
The pictures you see here show how h shrinks. This is the graphical version of taking the limit as h goes to zero. During this process, "y" doesn't change.

$\lim\limits_{h\to 0}\dfrac{f(x+h,y)-f(x,y)}{h}$ This is the difference quotient we're going to evaluate.

Our function is $f(x,y)=2x+y$.
$f(x+h,y)=2(x+h)+y$=value of function at $(x+h,y)$
$f(x,y)=2x+y$=value of function at (x,y)
$f(x+h,y)-f(x,y)=2(x+h)+y-(2x+y)=2x+2h+y-(2x+y)=2x+2h+y-2x-y=2h$
2h is the differece in the values of the function. So now we can divide this by h, $\dfrac{2h}{h}=2$. This calculation confirms that the rate of change is 2.

8)

Now we're going to visualize taking a limit in a physical sense. Below is a rectangle. First, as before, we write a function to represent the perimeter as P(x,y)=2x+2y. Now we're going to find the rate of change with respect to x. This means x changes, and y does not.



The pictures above illustrate how you should visualize letting h go to zero.

$$\lim_{h\to 0}\frac{P(x+h,y)-P(x,y)}{h}=\lim_{h\to 0}\frac{2(x+h)+2y-(2x+2y)}{h}=\lim_{h\to 0}\frac{2x+2h+2y-2x-2y}{h}=\lim_{h\to 0}\frac{2h}{h}=2$$

This calculation confirms that the limit of this difference quotient is 2. This means that the rate of change of the perimeter with respect to x is 2.

Amuse yourself by answering the following questions.

1) Draw a picture to represent $f(x,y)=3x+2y$
Hint: First close your eyes, and make sure you have a clear mental image of what to draw.

2) Draw slope triangles, and show in the picture the slope in the x direction.
Hint: First close your eyes, and make sure you have a clear mental image of what to draw.

3) Draw slope triangles, and show in the picture the slope in the y direction.
Hint: First close your eyes, and make sure you have a clear mental image of what to draw.

4) Draw a picture to represent $f(1,y)$ and mark the slope in the y direction.
Hint: First close your eyes, and make sure you have a clear mental image of what to draw.

5) Draw a picture to represent $f(4,y)$ and mark the slope in the y direction.
Hint: First close your eyes, and make sure you have a clear mental image of what to draw.

6) When we transition from $f(1,y)$ to $f(4,y)$, does the slope in the y direction change?
Hint: First close your eyes, and make sure you have a clear mental image of what to draw.

7) Draw a picture to represent $f(x,1)$ and mark the slope in the x direction.
Hint: First close your eyes, and make sure you have a clear mental image of what to draw.

8) Draw a picture to represent $f(x,2)$ and mark the slope in the x direction.
Hint: First close your eyes, and make sure you have a clear mental image of what to draw.

9) When we transition from $f(x,1)$ to $f(x,2)$, does the slope in the x direction change?
Hint: First close your eyes, and make sure you have a clear mental image of what to draw.

10) What's the meaning of $f(x,1)$? $f(x,2)$? $f(x,3)$? $f(x,4)$? Answer in pictures.
Hint: First close your eyes, and make sure you have a clear mental image of what to draw.

11) (Fill the blank).If the partial rate of change is given by -3 along x, then for every increase of _____ unit along _____, the value of the function changes by ______.

12) (Fill the blank).If the partial rate of change is given by 1.4 along y, then for every increase of _____ unit along _____, the value of the function changes by ______.

13) What's the meaning of $f(1,y)$? $f(2,y)$? $f(3,y)$? $f(4,y)$? Answer in pictures.
Hint: Close your eyes first, and imagine what you have to draw.

14) Draw a rectangle that is 2x wide, and 2y high. Write an expression for the perimeter.

15) Find the rate of change of the perimeter with respect to x.

16) Find the rate of change of the perimeter with respect to y.

17) Go back to the rectangle. Set $y=1$, and vary the x. Do you truly understand why we use the phrase "partial rate of change"? Do you have a truly clear mental image of the meaning of this phrase? If not, close your eyes now, and start using your imagination.

18) Are the rates of change in each direction constant or variable for the rectangle?
Can you truly and vividly imagine what this is asking?

19) Begin with $f(x,y)=3x-2y$. Now change the slope in the x direction, as shown below.
$f(x,y)=4x-2y$ Imagine and draw a picture
$f(x,y)=5x-2y$ Imagine and draw a picture
$f(x,y)=-3x-2y$ Imagine and draw a picture
Does changing the slope in the x direction change the slope in the y direction?

20) Begin with $f(x,y)=ax+3y$
What's $f(1,y)$?
What's f_a?
What's the effect of changing a on the orientation of the plane?
Does changing a have any impact on the slope along the y axis?