$\lim_{x \to 0^+} x^x$ We have 0^0

$y = x^x$ Rewrite as a function

$ln(y) = ln(x^x)$ Take the natural log of both sides

$ln(y) = xln(x)$ Bring the x down

$\lim_{x \to 0^+} ln(y) = \lim_{x \to 0^+} xln(x)$ Apply the limit

Find the limit on the right using the rule of L'Hopital $\frac{\infty}{\infty}$

$$\lim_{x \to 0^+} \frac{ln(x)}{x^{-1}} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x \to 0^+} \frac{1}{x} \cdot \frac{-x^2}{1} = \lim_{x \to 0^+} -x = 0$$

$$\lim_{x \to 0^+} ln(y) = 0$$

Exponentiate both sides.

$$e^{\lim_{x \to 0^+} ln(y)} = e^0$$

Because ln is continuous, bring the limit inside the function.

$$e^{ln\left(\lim_{x \to 0^+} y\right)} = 1$$

Because ln and e are inverses, just bring down the limit.

$$\lim_{x \to 0^+} y = 1$$

Replace y with its definition in terms of x.

$$\lim_{x \to 0^+} x^x = 1$$

$$\lim_{x \to \infty} x^{\frac{1}{x}}$$ Here we have ∞^0

$$y = x^{\frac{1}{x}}$$ Rewrite as a function

$$ln(y) = ln\left(x^{\frac{1}{x}}\right)$$ Take the natural log of both sides

$$ln(y) = \frac{1}{x}\, ln(x)$$ Bring down the $\frac{1}{x}$

$$\lim_{x \to \infty} ln(y) = \lim_{x \to \infty} \frac{ln(x)}{x}$$ Apply the limit

Find the limit on the right using the rule of L'Hopital because we have $\frac{\infty}{\infty}$

$$\lim_{x \to \infty} \frac{ln(x)}{x} = \lim_{x \to \infty} \frac{\frac{1}{x}}{1} = \lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} ln(y) = 0$$

Exponentiate both sides.

$$e^{\lim_{x \to \infty} ln(y)} = e^0$$

Because ln is continuous, bring the limit inside the function.

$$e^{ln\left(\lim_{x \to \infty} y\right)} = 1$$

Because ln and e are inverses, just bring down the limit. e gets rid of ln

$$\lim_{x \to \infty} y = 1$$

Replace y with its definition in terms of x.

$$\lim_{x \to \infty} x^{\frac{1}{x}} = 1$$

Watch the video version of this problem here:
http://www.tomsmath.com/calculus-library---hard-limits.html
Use THISISMAGNIFICENT as the password.

$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ Here we have 1^∞

$y=\left(1+\frac{1}{x}\right)^x$ Rewrite as a function

$ln(y)=ln\left(\left(1+\frac{1}{x}\right)^x\right)$ Take the natural log of both sides

$ln(y)=x\ ln\left(1+\frac{1}{x}\right)$ Bring down the x

$\lim_{x\to\infty} ln(y)=\lim_{x\to\infty} x\cdot ln\left(1+\frac{1}{x}\right)$ Apply the limit

Find the limit on the right using the rule of L'Hopital because we have $\infty\cdot\infty$
Here we have to use the chain rule on top and power rule on bottom.

$$\lim_{x\to\infty}\frac{ln\left(1+\frac{1}{x}\right)}{x^{-1}}=\lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}}\cdot\frac{-1}{x^2}}{\frac{-1}{x^2}}=\lim_{x\to\infty}\frac{1}{1+\frac{1}{x}}=\frac{\lim_{x\to\infty}1}{\lim_{x\to\infty}1+\lim_{x\to\infty}\frac{1}{x}}=1$$

$\lim_{x\to\infty} ln(y)=1$

Exponentiate both sides.

$e^{\lim_{x\to\infty} ln(y)}=e^1$

Because ln is continuous, bring the limit inside the function.

$e^{ln\left(\lim_{x\to\infty} y\right)}=e$

Because ln and e are inverses, just bring down the limit. e gets rids of ln.

$\lim_{x\to\infty} y=e$

Replace y with its definition in terms of x. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$

$\lim_{x \to 0^+} x^{x^2}$ We have 0^0

$y = x^{x^2}$ Rewrite as a function

$ln(y) = ln\left(x^{x^2}\right)$ Take the natural log of both sides

$ln(y) = x^2 \cdot ln(x)$ Bring down x^2

$\lim_{x \to 0^+} ln(y) = \lim_{x \to 0^+} x^2 \; ln(x)$ Apply the limit

Find the limit on the right using the rule of L'Hopital because we have $0 \cdot -\infty$

$$\lim_{x \to 0^+} \frac{ln(x)}{x^{-2}} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x \to 0^+} \frac{1}{x} \cdot \frac{-x^3}{2} = \frac{1}{2} \cdot \lim_{x \to 0^+} -x^2 = 0$$

$$\lim_{x \to 0^+} ln(y) = 0$$

Exponentiate both sides.

$$e^{\lim_{x \to 0^+} ln(y)} = e^0$$

Because ln is continuous, bring the limit inside the function.

$$e^{ln\left(\lim_{x \to 0^+} y\right)} = 1$$

Because ln and e are inverses, just bring down the limit.

$$\lim_{x \to 0^+} y = 1$$

Replace y with its definition in terms of x.

$$\lim_{x \to 0^+} x^{x^2} = 1$$

$\lim_{x \to 0^+} \cos(x)^{\frac{1}{x}}$ We have 1^{∞}

$y = \cos(x)^{\frac{1}{x}}$ Rewrite as a function

$\ln(y) = \ln\left(\cos(x)^{\frac{1}{x}}\right)$ Take the natural log of both sides

$\ln(y) = x^2 \cdot \ln(\cos(x))$ Bring down $\frac{1}{x}$

$\lim_{x \to 0^+} \ln(y) = \lim_{x \to 0^+} \frac{1}{x} \ln(\cos(x))$ Apply the limit

Find the limit on the right using the rule of L'Hopital because we have $\frac{0}{0}$

$$\lim_{x \to 0^+} \frac{\ln(\cos(x))}{x} = \lim_{x \to 0^+} \frac{\frac{1}{\cos(x)} \cdot -\sin(x)}{1} = \lim_{x \to 0^+} \frac{-\sin(x)}{\cos(x)} = \frac{-\sin(0)}{\cos(0)} = \frac{0}{1} = 0$$

$\lim_{x \to 0^+} \ln(y) = 0$

Exponentiate both sides.

$e^{\lim_{x \to 0^+} \ln(y)} = e^0$

Because ln is continuous, bring the limit inside the function.

$e^{\ln\left(\lim_{x \to 0^+} y\right)} = 1$

Because ln and e are inverses, just bring down the limit.

$\lim_{x \to 0^+} y = 1$

Replace y with its definition in terms of x. $\lim_{x \to 0^+} \cos(x)^{\frac{1}{x}} = 1$

Watch the video version of this problem here:
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Use THISISMAGNIFICENT as the password.

$$\lim_{x\to\infty}\left(1+\frac{P}{x}\right)^{x}$$ Here we have 1^{∞}

$$y=\left(1+\frac{P}{x}\right)^{x}$$ Rewrite as a function

$$ln(y)=ln\left(\left(1+\frac{P}{x}\right)^{x}\right)$$ Take the natural log of both sides

$$ln(y)=x\ ln\left(1+\frac{P}{x}\right)$$ Bring down the x

$$\lim_{x\to\infty} ln(y)=\lim_{x\to\infty} x\cdot ln\left(1+\frac{P}{x}\right)$$ Apply the limit

Find the limit on the right using the rule of L'Hopital because we have $\infty\cdot\infty$
Here we have to use the chain rule on top and power rule on bottom.

$$\lim_{x\to\infty}\frac{ln\left(1+\frac{P}{x}\right)}{x^{-1}}=\lim_{x\to\infty}\frac{\frac{1}{1+\frac{P}{x}}\cdot\frac{-1\cdot P}{x^{2}}}{\frac{-1}{x^{2}}}=\lim_{x\to\infty}\frac{P}{1+\frac{P}{x}}=\frac{\lim_{x\to\infty} P}{\lim_{x\to\infty} 1+\lim_{x\to\infty}\frac{P}{x}}=P$$

$$\lim_{x\to\infty} ln(y)=P$$

Exponentiate both sides.

$$e^{\lim_{x\to\infty} ln(y)}=e^{P}$$

Because ln is continuous, bring the limit inside the function.

$$e^{ln\left(\lim_{x\to\infty} y\right)}=e^{P}$$

Because ln and e are inverses, just bring down the limit. e gets rids of ln.

$$\lim_{x\to\infty} y=e^{P}$$

Replace y with its definition in terms of x. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e^{P}$

Correction: Instead of 1 over x, it should be P over x in the last line.

Watch the video version of this problem here:
http://www.tomsmath.com/calculus-library---hard-limits.html
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$\lim_{x \to 0^+} \sin(x)^{\sin(x)}$ Here we have 0^0

$y = (\sin(x))^{\sin(x)}$ Rewrite as a function

$\ln(y) = \ln\left(\sin(x)^{\sin(x)}\right)$ Take the natural log of both sides

$\ln(y) = \sin(x)\ \ln(\sin(x))$ Bring down the $\sin(x)$

$\lim_{x \to 0^+} \ln(y) = \lim_{x \to 0^+} \sin(x) \cdot \ln(\sin(x))$ Apply the limit

Find the limit on the right using the rule of L'Hopital because we have $0 \cdot -\infty$

$$\sin(x) = \frac{1}{\frac{1}{\sin(x)}} = \frac{1}{\csc(x)}$$

$$\lim_{x \to 0^+} \frac{\ln(\sin(x))}{\csc(x)} = \lim_{x \to 0^+} \frac{\frac{1}{\sin(x)} \cdot \cos(x)}{-\cot(x) \cdot \csc(x)} = \lim_{x \to 0^+} \frac{\cot(x)}{-\cot(x) \cdot \csc(x)} = \lim_{x \to 0^+} \frac{1}{-\csc(x)} = \lim_{x \to 0^+} -\sin(x) = 0$$

$$\lim_{x \to 0^+} \ln(y) = 0$$

Exponentiate both sides.

$$e^{\lim_{x \to 0^+} \ln(y)} = e^0$$

Because ln is continuous, bring the limit inside the function.

$$e^{\ln\left(\lim_{x \to 0^+} y\right)} = 1$$

Because ln and e are inverses, just bring down the limit. e gets rids of ln.

$$\lim_{x \to 0^+} y = 1$$

Replace y with its definition in terms of x. $\lim_{x \to 0^+} \sin(x)^{\sin(x)} = 1$

$\lim_{x \to 1^+} x^{\frac{1}{x-1}}$ We have 1^{∞}

$y = x^{\frac{1}{x-1}}$ Rewrite as a function

$ln(y) = ln\left(x^{\frac{1}{x-1}}\right)$ Take the natural log of both sides

$ln(y) = \frac{1}{x-1} ln(x)$ Bring down $\frac{1}{x-1}$

$\lim_{x \to 1^+} ln(y) = \lim_{x \to 1^+} \frac{ln(x)}{x-1}$ Apply the limit

Find the limit on the right using the rule of L'Hopital because we have $\frac{0}{0}$

$$\lim_{x \to 1^+} \frac{ln(x)}{x-1} = \lim_{x \to 1^+} \frac{\frac{1}{x}}{1} = \lim_{x \to 1^+} \frac{1}{x} = 1$$

$$\lim_{x \to 1^+} ln(y) = 1$$

Exponentiate both sides.

$$e^{\lim_{x \to 1^+} ln(y)} = e^1$$

Because ln is continuous, bring the limit inside the function.

$$e^{ln\left(\lim_{x \to 1^+} y\right)} = e$$

Because ln and e are inverses, just bring down the limit.

$$\lim_{x \to 1^+} y = e$$

Replace y with its definition in terms of x. $\lim_{x \to 1^+} x^{\frac{1}{x-1}} = e$

$$\lim_{x \to 1^+} x^{\frac{1}{1-x}}$$

$y = x^{\frac{1}{1-x}}$ Rewrite as a function

$ln(y) = ln\left(x^{\frac{1}{1-x}}\right)$ Take the natural log of both sides

$ln(y) = \frac{1}{1-x} ln(x)$ Bring down

$$\lim_{x \to 1^+} ln(y) = \lim_{x \to 1^+} \frac{ln(x)}{1-x}$$

Find the limit on the right using the rule of L'Hopital because we have

$$\lim_{x \to 1^+} \frac{ln(x)}{1-x} = \lim_{x \to 1^+} \frac{\frac{1}{x}}{-1} = \lim_{x \to 1^+} \frac{-1}{x} = -1$$

$$\lim_{x \to 1^+} ln(y) = -1$$

Exponentiate both sides.

$$e^{\lim_{x \to 1^+} ln(y)} = e^{-1}$$

Because ln is continuous, bring the limit inside the function.

$$e^{ln\left(\lim_{x \to 1^+} y\right)} = e^{-1}$$

Because ln and e are inverses, just bring down the limit.

$$\lim_{x \to 1^+} y = e^{-1}$$

Replace y with its definition in terms of x. $\lim_{x \to 1^+} x^{\frac{1}{1-x}} = e^{-1} = \frac{1}{e}$

$\lim_{x \to \infty} (ln(x))^{\frac{1}{x}}$ Here we have ∞^0

$y = (ln(x))^{\frac{1}{x}}$ Rewrite as a function

$ln(y) = ln\left(ln(x)^{\frac{1}{x}}\right)$ Take the natural log of both sides

$ln(y) = \frac{1}{x} ln(ln(x))$ Bring down the $\frac{1}{x}$

$\lim_{x \to \infty} ln(y) = \lim_{x \to \infty} \frac{1}{x} \cdot ln(ln(x))$ Apply the limit

Find the limit on the right using the rule of L'Hopital because we have $\frac{\infty}{\infty}$
Here we have to use the chain rule on top and power rule on bottom.

$$\lim_{x \to \infty} \frac{ln(ln(x))}{x} = \lim_{x \to \infty} \frac{\frac{1}{ln(x)} \cdot \frac{1}{x}}{1} = \lim_{x \to \infty} \frac{1}{ln(x) \cdot x} = 0$$

$$\lim_{x \to \infty} ln(y) = 0$$

Exponentiate both sides.

$$e^{\lim_{x \to \infty} ln(y)} = e^0$$

Because ln is continuous, bring the limit inside the function.

$$e^{ln\left(\lim_{x \to \infty} y\right)} = 1$$

Because ln and e are inverses, just bring down the limit. e gets rids of ln.

$$\lim_{x \to \infty} y = 1$$

Replace y with its definition in terms of x. $\lim_{x \to \infty} ln(x)^{\frac{1}{x}} = 1$

$$\lim_{x\to\infty} (1+2x)^{\frac{1}{3\ln(x)}}$$ Here we have ∞^0

$$y=(1+2x)^{\frac{1}{3\ln(x)}}$$ Rewrite as a function

$$\ln(y)=\ln\left((1+2x)^{\frac{1}{3\ln(x)}}\right)$$ Take the natural log of both sides

$$\ln(y)=\frac{1}{3\ln(x)}\ln(1+2x)$$ Bring down the $\frac{1}{3\ln(x)}$

$$\lim_{x\to\infty} \ln(y)=\lim_{x\to\infty} \frac{1}{3\ln(x)}\cdot\ln(1+2x)$$ Apply the limit

Find the limit on the right using the rule of L'Hopital because we have $\frac{\infty}{\infty}$
Here we have to use the chain rule on top.

$$\lim_{x\to\infty} \frac{\ln(1+2x)}{3\ln(x)}=\lim_{x\to\infty} \frac{\frac{1}{1+2x}\cdot 2}{\frac{3}{x}}=\lim_{x\to\infty} \frac{2}{1+2x}\cdot\frac{x}{3}=\frac{2}{3}\cdot\lim_{x\to\infty}\frac{x}{2x}=\frac{2}{3}\cdot\frac{1}{2}=\frac{1}{3}$$

1+2x is just like 2x when x is big

$$\lim_{x\to\infty} \ln(y)=\frac{1}{3}$$

Exponentiate both sides.

$$e^{\lim_{x\to\infty}\ln(y)}=e^{\frac{1}{3}}$$

Because ln is continuous, bring the limit inside the function.

$$e^{\ln\left(\lim_{x\to\infty} y\right)}=e^{\frac{1}{3}}$$

Because ln and e are inverses, just bring down the limit. e gets rids of ln.

$$\lim_{x\to\infty} y=e^{\frac{1}{3}}$$

Replace y with its definition in terms of x. $\lim_{x\to\infty} (1+2x)^{\frac{1}{3\ln(x)}}=e^{\frac{1}{3}}$

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65 Detailed Examples of Using the Chain Rule http://www.amazon.com/dp/B00HYO66ZS
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