

1) First transform the equation into standard form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

2) Find the integrating factor

$$I(x) = e^{\int P(x) dx}$$

3) Multiply the equation by the integrating factor

$$I(x) \frac{dy}{dx} + I(x) \cdot P(x)y = I(x) \cdot Q(x)$$

4) Rewrite the left side as it would look before the application of the product rule.

$$(I(x) \cdot y)' = I(x) \cdot Q(x)$$

5) Integrate both sides to get rid of the prime on the left

$$\int (I(x) \cdot y)' dx = \int I(x) \cdot Q(x) dx + C$$

$$I(x) \cdot y = \int I(x) \cdot Q(x) dx + C$$

6) Isolate y by dividing both sides by the integrating factor.

$$y = \frac{1}{I(x)} \left( \int I(x) \cdot Q(x) dx + C \right)$$