

Convert the equation $ydx - xdy = 0$ into exact form.

1) Identify M. This is usually the function in front of the dx: $M=y$

2) Differentiate M with respect to y: $M_y = \frac{\partial}{\partial y}(y) = 1$

3) Identify N. This is usually the function in front of dy: $N=-x$

4) Differentiate N with respect to x: $N_x = \frac{\partial}{\partial x}(-x) = -1$

5) Compare the results of steps 2) and 4) above.

$$M_y \neq N_x$$

This means the equation is not exact.

6) Form the integrating factor as shown below.

$$\frac{1}{M}(N_x - M_y) = \frac{1}{y}(-1 - 1) = \frac{1}{y} \cdot (-2) = \frac{-2}{y} \quad \text{This is a function of y only. It's } \frac{-2}{y}$$

$$\text{So the integrating factor is } e^{\int \frac{-2}{y} dx} = e^{-2 \ln(y)} = e^{\ln(y^{-2})} = y^{-2}$$

Remember that e and ln are inverses, so they cancel, leaving only $y^{-2} = \frac{1}{y^2}$

7) Multiply the original equation by the integrating factor.

$$\frac{1}{y^2}(ydx - xdy) = \frac{1}{y^2}(0)$$

$$\frac{y}{y^2}dx - \frac{1}{y^2} \cdot x dy = 0 \quad \text{Distribute into the terms}$$

$$\frac{1}{y}dx - \frac{x}{y^2}dy = 0 \quad \text{Simplify}$$

8) Now the equation is ready to be solved as is usual for exact equations.
