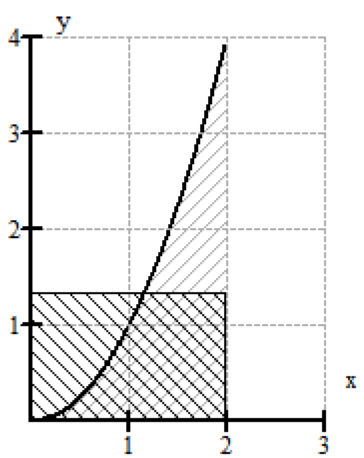
Dear Reader,
Thank you for considerting purchasing this booklet. This booklet provides you with 12 detailed and illustrated examples of finding the averages of various types of functions. Each step is shown, and each example is illustrated with a picture so you don't have a chance to forget what the meaning is of the number. As part of your purchase today, you also receive a link to a PDF version of the file, and access to a library of more than 420 HD math videos. The links are provided at the end of the file. In order to be successful in mathematics, you don't need a special gift. Rather, you simply have to practice the correct way, and within a short period of time, you'll have all the details internalized. This is the simple truth related to learning mathematics. I'm speaking as somebody who absolutely hated math as a child. But I've learned to represent it using pictures as much as possible, so you should find an equally effective way of learning that suits your personal style. Above all else, remember that persistence is more than enough 99% of the time to overcome all challenges. If you'll remain persistent, and apply yourself daily, you'll learn everything in short order.
Your Fellow Explorer,
Tom

1) $f(x)=x^2$ on $[0,2]$

$$f_{av}=\frac{1}{2-0}\int_0^2 x^2dx=\frac{1}{2}\left(\frac{x^{2+1}}{2+1}\right)_0^2=\frac{1}{2}\left(\frac{x^3}{3}\right)_0^2=\frac{1}{6}\left(x^3\right)_0^2=\frac{1}{6}\left(2^3-0^3\right)=\frac{1}{6}\left(2^3\right)=\frac{8}{6}=\frac{2\times 4}{2\times 3}=\frac{4}{3}$$

1) Apply the power rule to find the antiderivative.
2) Be sure to divide the integral by the length of the interval.
3) You can see the meaning of $\frac{4}{3}$ in the picture below. The area of the rectangle is the same as the area under the curve. The height of the rectangle is f_{av}

$$f_{av}(2)=\int_0^2 x^2dx$$



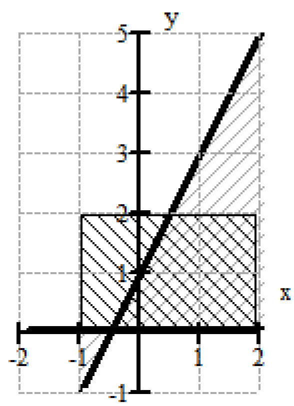
2) $f(x)=2x+1$ on $[-1,2]$

$$f_{av}=\frac{1}{2-(-1)}\int_{-1}^{2} 2x+1\,dx=\frac{1}{2+1}\int_{-1}^{2} 2x+1\,dx \;=\frac{1}{3}\left[x^2+x\right]_{-1}^{2}\;=\frac{1}{3}\left[2^2+2-((-1)^2+(-1))\right]$$

$$=\frac{1}{3}(4+2-(1-1))=\frac{1}{3}(6-(0))=\frac{1}{3}(6)=\frac{6}{3}=2$$

1) Apply the power rule to find the antiderivative.
2) Be sure to divide the integral by the length of the interval.
3) You can see the meaning of the "2" in the picture below. The area of the rectangle is the same as the area under the curve.
The height of the rectangle is f_{av}

$$f_{av}(2-(-1))=\int_{-1}^{2} 2x+1dx$$
$$f_{av}(2+1)=\int_{-1}^{2} 2x+1dx$$
$$f_{av}(3)=\int_{-1}^{2} 2x+1dx$$

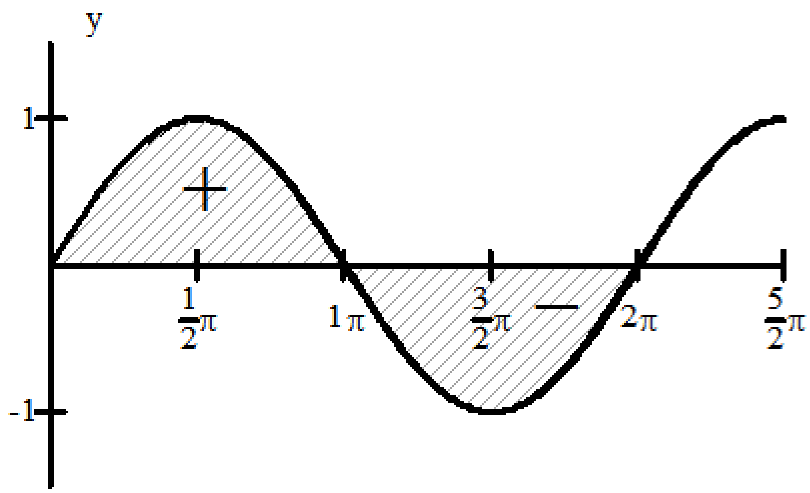


3) $f(x)=\sin(x)$ on $[0,2\pi]$

$$f_{av}=\frac{1}{2\pi-0}\int_0^{2\pi}\sin(x)dx=\frac{1}{2\pi}\int_0^{2\pi}\sin(x)\,dx=\frac{1}{2\pi}(-\cos(x))\Big|_0^{2\pi}=\frac{-1}{2\pi}(\cos(x))\Big|_0^{2\pi}$$

$$=\frac{-1}{2\pi}(\cos(2\pi)-\cos(0))=\frac{-1}{2\pi}(1-1)=\frac{-1}{2\pi}(0)=0$$

1) Apply the antiderivative rule for $\sin(x)$
2) Be sure to divide the integral by the length of the interval.
3) In the picture below we can see that the areas are equal, but of opposite sign, so they cancel.

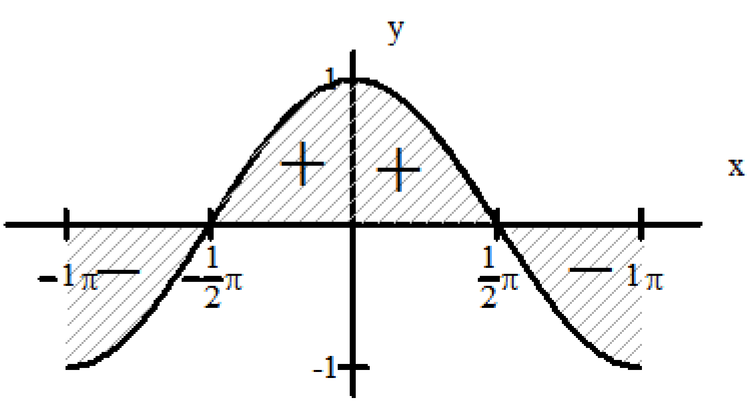


4) $f(x)=\cos(x)$ on $[-\pi,\pi]$

$$f_{av}=\frac{1}{\pi-(-\pi)}\int_{-\pi}^{\pi}\cos(x)dx=\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(x)\,dx=\frac{1}{2\pi}(\sin(x))_{-\pi}^{\pi}=\frac{1}{2\pi}(\sin(\pi)-\sin(-\pi))$$

$$=\frac{1}{2\pi}(0-0)=\frac{1}{2\pi}(0)=0$$

1) Apply the antiderivative rule for cos(x)
2) Be sure to divide the integral by the length of the interval.
3) In the picture below we can see that the areas are equal, but of opposite sign, so they cancel.

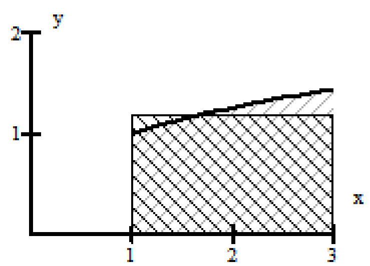


5) $f(x)=\sqrt[3]{x}$ on [1,3]

$$f_{av}=\frac{1}{3-1}\int_1^3 \sqrt[3]{x}\,dx=\frac{1}{2}\int_1^3 x^{\frac{1}{3}}\,dx=\frac{1}{2}\left(\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right)_1^3=\frac{1}{2}\left(\frac{x^{\frac{1}{3}+\frac{3}{3}}}{\frac{1}{3}+\frac{3}{3}}\right)_1^3=\frac{1}{2}\left(\frac{x^{\frac{4}{3}}}{\frac{4}{3}}\right)_1^3=\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(x^{\frac{4}{3}}\right)_1^3=\frac{3}{8}\left(x^{\frac{4}{3}}\right)_1^3$$

$$=\frac{3}{8}\left(3^{\frac{4}{3}}-1^{\frac{4}{3}}\right)=\frac{3}{8}\left(3^{\frac{4}{3}}-1\right)\approx 1.248$$

1) Rewrite the integrand so you can see how to apply the power rule
2) At the end, you can approximate the final result as 1.248
3) The area under the curve between 1 and 3 is the same as the area of the rectangle whose height is about 1.248



6) $f(x)=2x\sqrt{1+x^2}$ on $[0,2]$

$$f_{av}=\frac{1}{2-0}\int_0^2 2x\sqrt{1+x^2}\,dx=\frac{1}{2}\int_0^2 2x\sqrt{1+x^2}\,dx$$

At this point, you find the antiderivative using u–substitution.

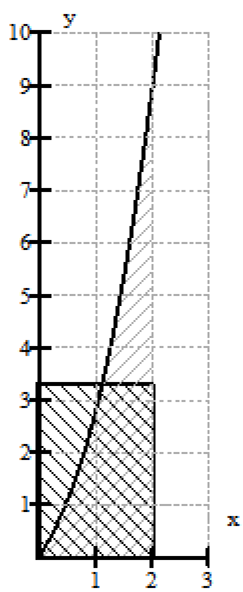
$u=1+x^2 \qquad du=2xdx$

Now replace in the integral, and drop the limits for now, and ignore the $\frac{1}{2}$

$$\int 2x\sqrt{1+x^2}\,dx=\int\sqrt{1+x^2}\,2xdx=\int\sqrt{u}\,du=\int u^{\frac{1}{2}}du=\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}=\frac{u^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}}=\frac{u^{\frac{3}{2}}}{\frac{3}{2}}=\frac{2}{3}u^{\frac{3}{2}}$$

Now you can multiply by the $\frac{1}{2}$ and replace u with its definiton, and apply the limits.

$$f_{av}=\frac{1}{2}\left(\frac{2}{3}\left(1+x^2\right)^{\frac{3}{2}}\right)_0^2=\frac{1}{3}\left(\left(1+(2)^2\right)^{\frac{3}{2}}-\left(1+(0)^2\right)^{\frac{3}{2}}\right)=\frac{1}{3}\left((1+4)^{\frac{3}{2}}-(1)^{\frac{3}{2}}\right)=\frac{1}{3}\left((5)^{\frac{3}{2}}-1\right)\approx 3.393$$

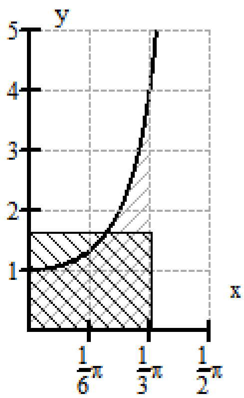


Remember that this tells us that the area of the rectangle is the same as the area under the curve. The height of the rectangle is equal to the average value of the function.

7) $f(x)=\sec^2(x)$ on $\left[0,\frac{\pi}{3}\right]$

$$f_{av}=\frac{1}{\frac{\pi}{3}-0}\int_0^{\frac{\pi}{3}}\sec^2(x)\,dx=\frac{1}{\frac{\pi}{3}}\int_0^{\frac{\pi}{3}}\sec^2(x)\,dx=\frac{3}{\pi}\int_0^{\frac{\pi}{3}}\sec^2(x)\,dx=\frac{3}{\pi}\left(\tan(x)\right)\Big|_0^{\frac{\pi}{3}}$$

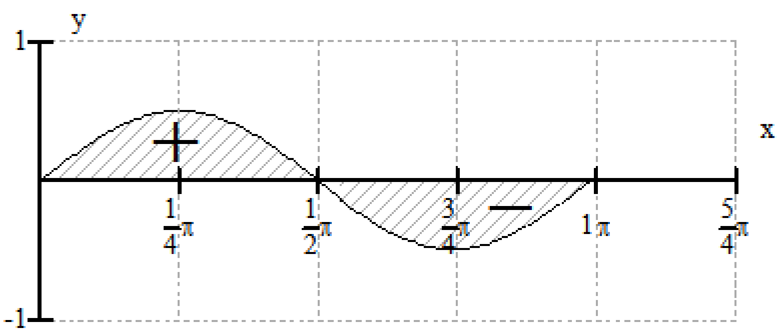
$$=\frac{3}{\pi}\left(\tan\left(\frac{\pi}{3}\right)-\tan(0)\right)=\frac{3}{\pi}(\sqrt{3}-0)=\frac{3\sqrt{3}}{\pi}$$



Remember this means that the area of the rectangle whose height is the average value of the function is the same as the area under the curve between 0 and $\frac{\pi}{3}$

8) $f(x)=\cos(x)\sin(x)$ on $[0,\pi]$

As the graph below shows, the two areas cancel.
This means that the average value of the function is 0.
$f_{av}=0$



9) Find c so that $f_{av}=3$ on $[0,c]$ for $f(x)=2x$.

This means we're searching for the upper limit of integration so that the average value of the function will be 3. Proceed as shown below.

$$3=\frac{1}{c-0}\int_0^c 2x\,dx$$

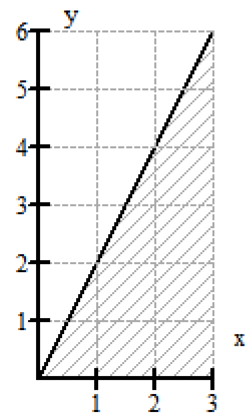
This equation states that we know the average value is 3, and we have to find the upper limit of integration.

$$3=\frac{1}{c}\int_0^c 2x\,dx$$

$$3=\frac{1}{c}\left(x^2\right)_0^c$$

Apply the power rule to find the antiderivative

$$3=\frac{1}{c}\left(c^2-0^2\right)$$



$$3=\frac{1}{c}\left(c^2\right)$$

$$\frac{c^2}{c}=\frac{c\times c}{c}=c$$

$$3=c$$

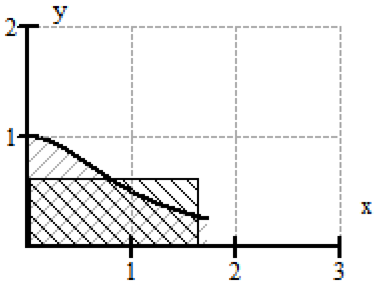
$$3=c$$

c=3 tells us that we must integrate from 0 to 3 in order to ensure that the average value of the function 2x on is 3.

10) $f(x)=\dfrac{1}{1+x^2}$ on $[0,\sqrt{3}]$

$$f_{av}=\frac{1}{\sqrt{3}-0}\int_0^{\sqrt{3}}\frac{1}{1+x^2}\,dx=\frac{1}{\sqrt{3}}\int_0^{\sqrt{3}}\frac{1}{1+x^2}\,dx=\frac{1}{\sqrt{3}}\left(\tan^{-1}(x)\right)_0^{\sqrt{3}}=\frac{1}{\sqrt{3}}\left(\tan^{-1}(\sqrt{3})-\tan^{-1}(0)\right)$$

$$=\frac{1}{\sqrt{3}}\left(\frac{\pi}{3}\right)=\frac{\pi}{3\sqrt{3}}$$



Remember this means that the area of the rectangle whose height is the average value of the function is the same as the area under the curve between 0 and $\sqrt{3}$

11) What average force must be applied over [0,2] to produce an impulse equivalent to that produced by f(t)=2t between [0,2]?

In physics, impulse is force applied over time. In this case, this means we have an integral on the right side, and on the left side we have the equivalent average force multiplied by the unit of time, which is 2. Once we have this setup, we can integrate the right side, and then divide by 2. This amounts to finding the average value of the force over the period of 2 units of time.

$$f_{av}(2)=\int_0^2 2t\,dt$$

$$f_{av}=\frac{1}{2}\int_0^2 2t\,dx$$ Divide both sids by 2

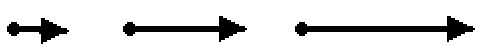
$$f_{av}=\frac{2}{2}\int_0^2 t\,dx$$ Factor the 2 from the integral

$$f_{av}=\int_0^2 t\,dx$$

$$f_{av}=\left(\frac{1}{2}t^2\right)_0^2$$ Apply the power rule to find the antiderivative

$$f_{av}=\frac{1}{2}\cdot 2^2=2$$

Now let's make sure we truly understand what this little number means. Imagine applying a variable force, over a period of 2 units of time. You can imagine this as an arrow that grows in length as the force increases.



f(1)=2 f(1.5)=3 f(2)=4

Our average value of 2 means we can replace the variable force at each instant in time with the average force, and still achieve the same impulse, as shown below.

12) Given $f(x)=2x$, find c so that $f(c)=f_{av}$ on $[0,4]$.

This means you must put 2c on the left, and and setup the integral for the average value of the function on the right.

$$2c=\frac{1}{4}\int_0^4 2x\,dx$$

Multiply both sides by $\frac{1}{2}$ and then use the power rule to integrate.

$$c=\frac{1}{2}\cdot\frac{1}{4}\int_0^4 2x\,dx=\frac{1}{8}\int_0^4 2x\,dx=\frac{2}{8}\int_0^4 x\,dx=\frac{2\times 1}{2\times 4}\int_0^4 x\,dx=\frac{1}{4}\left(\frac{1}{2}x^2\right)=\frac{1}{4}\left(\frac{1}{2}\cdot 4^2-0^2\right)=\frac{16}{8}=2$$

This value tells us that at x=2, the value of the function is equal to the average value of the function.