Find the tangent line to $f(x)=\cos(2x)$ at $x=\frac{\pi}{2}$

1) Because $\cos(2x)$ consists of the function $2x$ inside the cosine function, you have to use the chain rule. The chain rule applies when you plug one function into another.

2) The chain rule states that $\frac{d}{dx}f(g(x))=f'(g(x))g'(x)$

3) In our case, this means multiply the derivative of cosine by the derivative of $2x$

4) Now we can differentiate as follows:

$$f'(x)=\frac{d}{dx}\cos(2x)=-\sin(2x)(2)=-1\cdot 2\cdot \sin(2x)=-2\sin(2x)$$

5) Now we evaluate the derivative at $x=\frac{\pi}{2}$ to find the slope.

$$\begin{aligned} f'\left(\frac{\pi}{2}\right)&=-2\sin\left(2\left(\frac{\pi}{2}\right)\right) && \text{replace x with } \frac{\pi}{2} \\ &=-2\sin(\pi) && \text{cancel the 2's} \\ &=-2(0)=0 && \sin(\pi)=0 \end{aligned}$$

6) Now we use the equation $y-f(x_0)=f'(x_0)(x-x_0)$

In our case, $x_0=\frac{\pi}{2}$, so replace and simplify.

7) $y-\cos\left(2\left(\frac{\pi}{2}\right)\right)=0\left(x-\frac{\pi}{2}\right)$

$y-\cos(\pi)=0$

$y-(-1)=0$

$y+1=0$

$y=-1$

8) You can see $f(x)=\cos(2x)$ and the tangent line graphed below.